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Abstract

This project has been concerned with theoretical, algorithmic, and applied research in six areas of discrete applied mathematics. Work on graph theory and its applications has been concerned with graph coloring and stability, special classes of graphs, and graphs and discrete optimization. Work on discrete optimization has also dealt with location problems, preprocessing and decomposition, approximation, and applications of combinatorial optimization to nonlinear problems. Our research on posets and other combinatorial structures and their applications has been concerned with linear extensions and ideals, graphs and posets, posets and discrete optimization, and other useful combinatorial structures. Our effort in the area of computational complexity and efficient algorithms has concentrated on foundations of computational complexity and heuristics. Work on applications of discrete mathematics to decisionmaking has involved group decisionmaking, measurement and decisionmaking, and multiple conclusion logic. Our work on large scale scheduling problems has concentrated on the STORM I and STORM II models for routing aircraft, the aircrew scheduling problem, and the single base aircrews staging problem. Among the many applications we have considered are frequency assignment, task scheduling and air crew scheduling, location of warehouses and communication centers, maintenance problems, communications over noisy channels, and expert systems.

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to Air Force Office of Scientific Research

"A RUTCOR Project on Discrete Applied Mathematics"

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Accomplishments: August 1, 1985-September 30, 1988

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January 30, 1989

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A RUTCOR Project on Discrete Applied Mathematics

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SUMMARY OF RESEARCH ACCOMPLISHMENTS

September 30, 1987-September 30, 1988

This summary of research accomplishments is organized into the essentially the same sections and subsections as is our original proposal. Papers referred to by number are listed below in the list of publications prepared under the grant during the period September 30, 1987 to September 30, 1988.

## 1. Graph Theory and its Applications

The emphasis in our work in graph theory has been on questions relating to applications. It has been, and will continue to be, motivated by some basic problems in communications, transportation, scheduling, assignment, maintenance, and so on.

### 1.1. Graph Coloring and Stability

Much current work in graph theory is concerned with the related problems of finding optimal graph colorings and finding the largest stable set in a graph. Both of these problems are closely tied to practical applications, and our work on them so far has been connected to such applications.

In earlier years, we initiated the study of $T$-colorings of graphs in connection with frequency assignment problems. In such problems, the vertices of a graph $G$ represent transmitters and an edge between two vertices represents interference. We seek to assign to each vertex or transmitter $x$ a channel $f(x)$ over which $x$ can transmit, and for simplicity we take the channels to be positive integers. The assignment of channels is subject to the restriction that if two transmitters interfere, i.e., if the corresponding vertices are joined by an edge, then the channels assigned to these transmitters cannot be separated by a disallowed distance. To make this more precise, we fix a set $T$ of nonnegative integers and assign channels so that if vertices $x$ and $y$ are joined by an edge of $G$, then $|f(x)-f(y)|$ is not in $T$. (See Roberts [1986] for a summary of the literature of $T$-colorings and a statement of fundamental problems.) We have studied the problem of finding the optimum <u>span</u> of a $T$-coloring, the minimum separation between the smallest and largest $f(x)$ values. We have obtained in the thesis [72] a variety of results about $T$-colorings of complete graphs, the basic graphs to which most $T$-coloring problems can be reduced. We have also introduced in the thesis [72] and the paper [64] the beginnings of a theory of set $T$-colorings, variants of a theory of $T$-colorings in

which each transmitter receives more than one channel. This theory combines the n-tuple colorings of Gilbert [1972] and Stahl [1976] and the more general set colorings of Roberts [1979b] and Opsut and Roberts [1981, 1983a,b] with the T-colorings we have been studying. Some related work can be found in the paper by Furedi, Griggs and Kleitman [1988].

Graph colorings have a wide variety of applications in scheduling, fleet maintenance, traffic phasing, etc. (For a discussion of the applications of graph coloring, see Roberts [1984]. See also [64].) We have been working on some fundamental problems of graph coloring. In particular, we have studied list colorings of graphs. In many practical coloring problems, a choice of color to assign is restricted. A set or list of possible colors to be assigned to a vertex is specified, and we seek a graph coloring so that the color assigned to a vertex is chosen from its list. List colorings arise for instance in the channel assignment problem when we specify possible acceptable channels. Erdos, Rubin and Taylor [1979] introduced the idea of considering when a graph $G$ can be list-colored for every assignment of lists of $k$ colors in each list. If $G$ can always be list colored for every such asignment, we say that $G$ is k-choosable, and call the choice number of $G$ the smallest such $k$. Tesman [71,72] has calculated the choice number for chordal graphs. He has also introduced a similar theory of list colorings which are also T-colorings and obtained useful bounds on a T-choice number for T-colorings of chordal graphs.

We have also introduced ([37]) a theory of "hard to color graphs" in connection with approximate coloring algorithms. For a given approximate coloring algorithm, a graph is said to be slightly hard to color if some implementation of the algorithm uses more colors than the minimum needed. Similarly, a graph is said to be hard to color if every implementation of the algorithm results in a non-optimal coloring. We have shown that for the widely-used largest-first algorithm, there is a unique smallest hard-to-color graph.

We have been studying on-line algorithms for a variety of coloring problems. An algorithm is on-line if, as is the case in many practical problems, it is necessary to make choices as new data becomes available, rather than waiting until all of the problem is specified in advance. In earlier years, we started developing on-line algorithms for optimal ordinary colorings and optimal T-colorings. In the present year, the former effort has continued with the refinement and publication of paper [59]. This effort is described in detail in Section 4.3 below.

Turning to stable sets, we note that some well-known results of Turan [1941] in extremal graph theory, modified by Brigham and Dutton [1985], give a nice formula for a sharp lower bound on the number of edges of a graph of $n$ vertices and stability number $\alpha$. Hansen [1975], [1979] has given sharp lower and upper bounds on $\alpha$ given $n$ and $m$, the number of edges of the graph. In paper [45] we obtain sharp lower and upper bounds on $n$ given $m$ and $\alpha$.

## 1.2. Special Classes of Graphs

Many graph theory problems are extremely difficult when looked at in general, but turn out to be tractable when restricted to a special class of graphs. Hence, research in graph theory has in recent years emphasized the study of rich and interesting special classes of graphs, many of which arise from applications, and for which efficient algorithms can often be found to solve important optimization questions. Our work on special classes of graphs has reflected this point of view.

Among the classes of graphs we have studied are the threshold graphs. These graphs have connections to Guttman scaling in measurement theory and to synchronizing parallel processors. Of great importance in the study of threshold graphs has been the sequences of degrees of its vertices; a sequence of numbers arising this way is called a threshold sequence. Paper [63] studies the convex hull $D_n$ of the set of all degree sequences of length $n$ of arbitrary graphs, and shows that the threshold sequences are exactly the extreme points of $D_n$. A variety of related results about threshold sequences are obtained, for instance that every degree sequence is a convex combination of threshold sequences that are rearrangements of each other.

An important class of graphs with regard to applications is the class of competition graphs and its variants. These graphs, introduced by Cohen [1968], arise in communications over noisy channels; in the channel assignment problem mentioned above (which often is concerned with coloring a competition graph); in large-scale computer models of complex systems; and in the study of food webs in ecology. (See the surveys by Raychaudhuri and Roberts [1985] and by Lundgren [1989].) Recently, Scott [1987] introduced the notion of a competition-common enemy graph (CCE graph), a graph obtained from an acyclic directed graph by taking an edge between two vertices if and only if they have incoming arcs from two common vertices and outgoing arcs to two common vertices. Kim [53] has solved a problem of Seager in showing that every bipartite graph with one of its classes having size at most 4 is a CCE graph if at most two isolated vertices are added. In the process, she has found an interesting result about matrices which, under row and column permutations, are free of the pattern 1 0 1 on diagonals. Further results are in the paper [56]. Kim, McKee, McMorris, and Roberts [54] have developed a theory of p-competition graphs, graphs which arise from digraphs by taking an edge between two vertices if and only if they have outgoing arcs to at least $p$ common vertices. This theory falls into a more general theory of tolerance graphs (see Jacobson, et al. [1988]). Also in the study of competition graphs, we have solved a problem of Harary by calculating the largest competition number of a graph of $n$ vertices. (See Kim [53].) We have also proved ([53], [55]) a modified version of the conjecture of Opsut [1982] that if $G$ is a graph with the

property that each vertex has a neighborhood which can be covered by at most two cliques, then G has competition number at most 2. We have also obtained (see paper [46]) some further results on the most important open problem in the field of competition graphs, namely in identifying what acyclic digraphs have the property that their competition graphs are interval graphs. Following on our approach to this problem begun last year, we have attacked it by restricting the outdegree and indegree in the digraphs. Under degree restrictions, we have characterized the competition graphs and the acyclic digraphs whose competition graphs are interval graphs, and also the competition graphs which are interval graphs. The results make heavy use of the theory of combinatorial designs.

In other work which is related to combinatorial designs, we have studied the special class of graphs consisting of the complete graphs with an even number of vertices. A <u>$\lambda$-hyperfactorization</u> of $K_{2n}$ is a collection of 1-factors for which each pair of disjoint edges appears in precisely $\lambda$ of the 1-factors. Such a $\lambda$-hyperfactorization is called <u>trivial</u> if it contains each 1-factor of $K_{2n}$ with the same multiplicity and <u>simple</u> if each 1-factor appears at most once. Cameron [1976] and Jungnickel and Vanstone [1987] had found examples of nontrivial $\lambda$-hyperfactorizations for special values of $n$. In paper [11], we show the existence of nontrivial, simple $\lambda$-hyperfactorizations of $K_{2n}$ for all $n \geq 5$.

We have completed work started in previous years in which we have found an exact formula for the smallest number of edges of a graph of $n$ vertices which does not contain an induced forest of size $k$. (See [1].)

We have also studied a variety of intersection graphs, in particular interval graphs and indifference graphs, in paper [66]. These graphs have very important applications in scheduling problems, maintenance problems, traffic phasing problems, computer storage problems, task assignment problems, etc. The paper [66] surveys some of these applications and recent results about these important classes of graphs.

### <u>1.3. Graphs and Discrete Optimization</u>

A rather large effort has been devoted so far to discrete optimization, and we discuss this in detail in Section 2. Here, we describe some of our work which relates graph theory to discrete optimization. In applications of graph theory to problems of communication, one of the most important open problems remains the famous problem of Shannon which is to compute the (zero-error) capacity of a graph. In connection with this problem, Lovasz [1979] introduced orthogonal representations of graphs to compute the capacity of the 5-cycle, a well-known and important special case of Shannon's question. Grotschel, Lovasz, and Schrijver [1986] showed that orthogonal representations are intimately related to the vertex

packing polytope, and Grotschel, Lovasz, and Schrijver [1984] used orthogonal representations to design polynomial-time algorithms for finding maximum cliques and optimum colorings in perfect graphs. In paper [58], Lovasz, Saks, and Schrijver study the minimum dimension of the space in which orthogonal representations with certain non-degeneracy properties exist. They prove that a graph is k-connected if and only if its vertices can be represented by real vectors of length $n-k$ such that nonadjacent vertices are represented by orthogonal vectors and any $n-k$ of them are independent. They also show that the closure of the set of all representations with these properties is irreducible as an algebraic variety.

Paper [16] studies bidirected graphs, multigraphs in which every arc has either two tails, or two heads, or a tail and a head. Bidirected graphs have been considered in connection with such concepts of discrete optimization as matching polyhedra (Edmonds and Johnson [1970]) and network flows (Lawler [1976]). A bidirected graph is called <u>simple</u> if for each pair of vertices, there is at most one arc with those as endpoints. The <u>adjoint</u> of a bidirected graph $B$ has for vertices the arcs of $B$ and has an edge connecting two vertices if and only if the corresponding arcs are consecutive in $B$. A graph is quadratic (respectively <u>quadratic primitive</u>) if it is the adjoint of some bidirected (respectively simple bidirected) graph. We prove that the recognition problem for quadratic primitive graphs is NP-complete. The results extend to another class of quadratic graphs, the so-called partition quadratic graphs.

A <u>matching</u> in a graph is a collection of edges or complete 2-vertex subgraphs which have no common endpoints. Maximal matchings on graphs and weighted graphs arise in a wide variety applications in optimization problems which include job assignments, storage of computer programs, real estate transactions, etc. They had a classic application to pilot assignments for the Royal Air Force during World War II. We have studied in paper [21] a generalization of a matching called an <u>odd chain packing</u>, a collection of edge-disjoint chains of odd length such that all endpoints of these chains are distinct. We extend the augmenting chain theorem of matchings to odd chain packings and find an analogue of matching matroids. We show that we may restrict ourselves to packings by chains of lengths one or two, and obtain a min-max result for such packings for the special case of trees.

## 2. Discrete Optimization.

Discrete optimization problems arise in a large variety of vitally important practical scheduling, allocation, planning, and decisionmaking problems. Such practical problems have been one of the reasons that discrete optimization has become one of the most rapidly developing fields of mathematical programming. Another reason is that more and more mathematical fields (e.g., group theory, number theory, boolean algebra, graph theory, and polyhedral combinatorics) are becoming involved in the study of such optimization problems. We have

spent a great deal of time on discrete optimization questions in the past year.

### 2.1. Location Problems

Location problems arise whenever a large set of potential sites for placing certain units is available and a selection must be made of the sites to be utilized. Such problems arise naturally in situations like placing warehouses, satellites, communication centers, military units, or emergency services. We have studied a variety of location problems and approaches to solving them.

In paper [20], in work begun in earlier years, we have concluded our study of the classic simple plant location problem by formulating it as the minimization of a pseudo-Boolean function (see definition in Section 2.2). This formulation is transformed into two discrete optimization problems: A set covering problem and a weighted vertex packing problem on a graph. These three formulations of the problem are compared to similar formulations that have appeared in the literature and to the standard integer programming formulation.

Traditionally, location theory has studied where to locate desirable facilities. During the last decade, as increasing attention was being paid to environmental problems, the location of obnoxious facilities has been studied with increasing intensity. Much recent work focuses on the location of mutually obnoxious facilities. This leads to dispersion problems. For example, the p-maxisum dispersion problem consists of locating p facilities at vertices of a network in order to maximize the sum of the distances between all pairs of them. (See Kuby [1987].) In paper [42], we have shown the decision version of this problem to be strongly NP-complete. For a tree network, a solution algorithm with a complexity linear in the number of vertices is proposed for any given p. Moreover, the problem on a general network is shown to be a particular case of the quadratic knapsack problem, for which fairly efficient algorithms are available.

In paper [39] we study the continuous p-median problem for a network. We show that the sets of vertices and midpoints of edges of a network always contains a continuous median and that this extends to the case of the p-median problem. We provide linear algorithms for solving the 1-median problem on a tree.

The main body of facility location theory concentrates on the location of facilities under the control of a single decisionmaker. In contrast, in earlier years we considered the problem of locating a facility with respect to other competing facilities under the control of independent decisionmakers. We also considered the problem of locating facilities as the result of a collective action in which "clients" pursue their own interest within the mutual dependency imposed by a voting rule. Our foundational work on these two problems (paper [38]) has been revised for publication in the past year.

Finally, we have considered in paper [29] the problem of locating

several facilities among a given set of possible locations to maximize profit assuming that the pricing policy of a firm is either discriminatory prices (each client is given a separate price), uniform delivered prices (prices are the same to all), or uniform mill price (price at each factory is the same). We have found algorithms to solve the optimal location problem for all three cases and compare the results. The results show that a number of statements in the economic literature about the advantages of competition are not accurate.

## 2.2. Preprocessing and Decomposition

Discrete optimization problems are frequently too hard to solve. This can be because there are huge numbers of redundant variables present in their original formulation, because the coefficients in the constraints are disproportionately large, because there are unnecessary nonlinearities, etc. A frequently useful approach is to transform a given problem into a more structured one, or a small number of more structured problems, for which good solution methods exist. Our research effort has continued to emphasize such preprocessing of discrete optimization problems.

In our work on preprocessing and decomposition, we have continued to devote considerable effort to the set covering problem, one of the most fundamental, yet difficult problems studied in 0-1 linear programming. This problem has applications to crew scheduling, network disconnection, information retrieval, delivery problems, etc. Our work on set covering problems has continued to concentrate on defining and studying classes of such problems which can be decomposed into a small number of specially structured problems, for which good solution methods exist. This idea led us, in work cited in our previous two Interim Scientific Reports, to the study of bimatroidal independence systems, independence systems which can be expressed as the union of exactly two matroids. In Crama and Hammer [1986] and Crama and Hammer [1987], we found a family of independence systems that admit a unique decomposition into two prime matroidal components and such that the number of circuits of one of the components is exponentially large in the number of circuits of the original system. We also showed how to obtain a bimatroidal decomposition into prime components in polynomial time for a special class of independence systems generalizing the independence systems of graphs which had been studied earlier by Benzaken and Hammer. We have shown that in general, when the independence system is described by an "independence oracle," any algorithm for determining bimatroidalness performs in the worst case in an exponential number of steps. However, we have found a polynomial-time algorithm for the set-covering version of the problem (where the input consists of the complete list of circuits of the system). In paper [15], we have now been able to improve these results.

A pseudo-Boolean function is a real-valued function on $\{0,1\}^n$, and a Boolean function is a 0-1-valued pseudo-boolean function. Such functions have a wide variety of practical applications. As part of

our study of discrete optimization problems, we have been investigating the formulation of such problems using pseudo-boolean functions. Many times, boolean and pseudo-boolean methods allow the considerable simplification of combinatorial optimization problems expressed by using boolean and pseudo-boolean functions. We have already described in Section 2.1 our work on formulating the simple plant location problem as the minimization of a pseudo-Boolean function. To describe other work, let us note that a pseudo-Boolean function has a unique expression as a multilinear polynomial in $n$ variables. It is called almost-positive if all the coefficients in that expression, except maybe those in the linear part, are nonnegative. The almost-positive functions form a convex cone, given explicitly by its extreme rays. In work which was begun in earlier years (paper [17]), we describe this cone by a system of linear inequalities, which can be viewed as a natural generalization of supermodularity to higher orders. We also point out a characterization in terms of the sign of partial derivatives. Finally, in paper [19] we reconsider the classic algebraic method of Hammer and Rudeanu for solving pseudo-Boolean programming problems. We show that this method is linear for a particular class of pseudo-Boolean functions for which the co-occurrence graph is a partial $k$-tree. Some steps of the algorithm are revised and computational experience allows us to solve problems with $k \leq 10$ and at most 200 variables.

## 2.3. Approximation

A major theme in discrete mathematics in recent years has been the effort at finding methods for approximating solutions to problems and at finding exact solutions by successive approximations. The approximation problem has continued to be a main focus of our efforts so far.

We have already described in Section 1.1 our efforts at understanding what graphs are hard to color by any implementation of an approximation algorithm for graph coloring.

One approach to approximation is to associate with a given problem a "relaxation" of it, i.e, an easy problem the solution of which provides information about the solution of the original problem. Hammer, Hansen, and Simeone [1984] associated a linear program to a discrete optimization problem and showed that it provides a bound to the original one and fixes the optimal values of some of the variables. This technique is called roof duality. In the previous two years, we had begun to build on this fundamental notion of roof duality to study quadratic and more general polynomial 0-1 optimization problems, which have applications in the selection of R&D projects, of petroleum leases upon which to bid, of items to be included in any volume-limited or weight-limited space (the "knapsack" problem), and of routes to be served by a commercial or military carrier. In paper [26], we have investigated the roof duality gap,

the gap between the optimal value of the roof dual and of the original quadratic function in 0,1 variables whose optimal value is being approximated. We obtain a bound on this roof duality gap and show that in a special case where the off-diagonal elements of the Hessian matrix are nonnegative, the optimal value of the roof dual coincides with the so-called concave envelope. In paper [41], which has been revised and updated from a paper prepared last year, we have investigated the approximation using paved duality and have shown that in the general nonlinear case, paved-duality combined with standard linearization leads to the same bound as roof duality.

The notion of roof dual has also arisen in our investigation of the Separation Problem in paper [75]. Given a set of $n$ vertices with distances $d_{ij}$ between them defined, this problem is the problem of partitioning the vertices into two sets so as to maximize the total distance between them. It is shown that in the general case where the "distances" are symmetric, but not necessarily non-negative, the problem is equivalent to the 0,1 unconstrained quadratic optimization problem, and, in fact, it is equivalent to the general unconstrained Boolean optimization problem of degree 2. Linearization constructions based on the early work of Rhys [1970] lead to an integer linear program whose optimal solutions are optimal solutions to the Separation Problem; solutions to its relaxation are therefore upper bounds for the Separation Problem. It is shown that the dual to this linear program is exactly the roof dual. A preliminary heuristic approach to numerical solution is briefly investigated, and some very preliminary computational results are presented.

In other work begun in previous years, we have been examining how bad the gap can be between the linear and integer programming solutions to problems with 0-1 coefficients and constraints and we investigated a special class of problems by comparing matchings with fractional matchings. We solved the fractional version of the celebrated Erdos-Faber-Lovasz conjecture. Specifically, we proved that if $\mathcal{H}$ is a nearly disjoint hypergraph on $n$ vertices (i.e., a hypergraph with any two sets intersecting in at most one point) and $\mathcal{M}$ is the set of matchings of $\mathcal{H}$, then there is a function $w$ from $\mathcal{M}$ into the positive reals so that for all $A$ in $\mathcal{H}$, $\sum_{A \in M} w(M) \geq 1$ and $\sum_{M \in \mathcal{M}} w(M) \leq n$. This work has now been carefully written up as paper [51].

For a given optimization problem P considered as a function of the data, its marginal values are defined as the directional partial derivatives of the value of P with respect to perturbations in that data. For linear programs, formulas for a complete marginal value analysis were given by Mills [1956] and extended by Williams [1963]. Although these general formulas for complete marginal analysis of linear programming models have been known for some time, their application still generally seems to be restricted to the case of perturbing a single right hand side value when the dual optimal solution is unique. We present in paper [74] a number of instances in

which the more general formulas are of practical use in analyzing linear programming models and their solutions. Emphasis is on analysis and conclusions that can be made just on the basis of information ordinarily made available when a linear program is solved on a typical commercial code, i.e., a single primal optimal and single dual optimal solution.

In other work on the sensitivity of linear programs to changes in parameters, we have investigated in paper [40] the tolerance approach, an approach which allows us to determine by how much a set of parameters can simultaneously change either upwards or downwards while leaving the same basic solution optimal. In contrast, in the usual sensitivity analysis, one just changes one coefficient at a time or changes all coefficients in the same direction, governed by one parameter, which is very restrictive. In this paper, we apply this tolerance approach to the case where the coefficients are weights of the multiple linear objectives.

## 2.4. Applications of Combinatorial Optimization to Nonlinear Problems

In operations research, one makes the distinction between algorithms designed to find a local optimum and algorithms designed to find the global optimum. The vast majority of nonlinear programming algorithms belong to the first category, but increasing attention is being devoted to the latter one. We have found that many of the ideas underlying algorithms for combinatorial optimization can be transposed to the field of global optimization. Many practical problems in the engineering literature can be looked at as constrained global optimization problems. The most successful approaches to constrained global optimization appear to be interval analysis (Moore [1979], Hansen [1979, 1980], and Hansen and Sengupta [1981]); monotonicity analysis (Wilde [1978], Papalambros and Wilde [1979, 1980]); and computer algebra (Stoutemeyer [1975]). These three approaches have been combined and extended in a series of papers [32,33,36] we prepared last year and modified in the current year.

This year, our work on global optimization has continued. In particular, in paper [35], we have studied Piyavskii's algorithm which maximizes univariate functions $f(x)$ satisfying a Lipschitz condition. We have compared the numbers of iterations needed to obtain a bound within $\epsilon$ of the (unknown) optimal solution with a best possible algorithm ($n_B$) and with Piyavskii's method ($n_P$). The main result is that $n_P \leq n_B+1$ and that this bound is sharp. Several related bounds are obtained.

We have also found in paper [34] the global minimum of a univariate function by approximating this function piecewise by polynomials, which can then be minimized analytically. This corrects and generalizes some previous results by Wingo [1985] on minimization

of polynomials.

## 3. Posets and Other Combinatorial Structures and their Applications

Combinatorial structures such as matroids, graphs, block designs, and partially ordered sets (posets) have a wide variety of applications to practical problems. Our work on such combinatorial structures to date has emphasized graphs (see Section 1) and posets. We have, however, also encountered a variety of other structures in our work. In this section, we describe the work on posets and on these other combinatorial structures, with an emphasis on posets.

### 3.1. Linear Extensions and Ideals

Posets are among the fundamental objects of discrete mathematics. They have applications to the theory of computation, optimization, game theory, preference and decisionmaking, etc. Among the concepts of posets especially important in problems of searching and sorting are the concepts of linear extension and antichain. The dimension of a poset is the smallest number of linear extensions whose intersection is the poset. A well-known result of Hiraguchi [1955] states that the size of a poset is at least twice its dimension. The analogous question for lattices was posed by B. Sands. We have shown that the dimension of a projective plane of order $n$ is at least on the order of $n/\log n$, so the size of a lattice can be as small as $d^2 \log d$, where $d$ is the dimension. This work, begun in earlier years, has been updated and published. See paper [24].

In other work begun in an earlier year, we have been studying the shadows of antichains in the Boolean algebra $B(n)$, the sets covered by such antichains, and asking whether it is possible for such a shadow to contain a positive fraction of all the sets. K. Engel had originally asked how large such a shadow could be, and made the (natural) conjecture that it is $O(2^n/\sqrt{n})$, i.e., about the number of $n/2$-sets. Furedi and Kahn had disproved this earlier. In paper [25], we obtain the surprising result that it is possible for a shadow to have a positive fraction (more than a tenth) of all the sets. Remarkably, there is still no upper bound on the size of the shadow which is asymptotically less than $2^n$.

In studying linear extensions of posets, Kahn and Saks [1984] proved the very deep result that if a poset is not a chain, then for every $x$ and $y$, the probability that $x < y$ in an extension is bounded away from both 0 and 1, in particular by 3/11 and 8/11. By making use of the Brunn-Minkowski Theorem, we have now shown (paper [49]) a similar result with bounds $1/2e$ and $1-(1/2e)$. The bounds are not as good as the earlier ones, but the proof is much easier.

### 3.2. Graphs and Posets

One can sometimes learn something about a graph by associating with it a poset and one can sometimes learn something about a poset by associating with it a graph. In paper [69], begun last year, we have used such considerations to find an exact answer for the minimum number of pairwise comparisons needed to identify the larger class of a finite set which is divided into two as yet unknown classes.

### 3.3. Posets and Discrete Optimization

Posets play an important role in the analysis of a number of discrete optimization problems. We have prepared two papers, Hammer and Liu [27,28], which describe the important role of order relations in solving mathematical programming problems.

### 3.4. Other Useful Combinatorial Structures and their Applications

Other combinatorial structures have played an important role in our work from time to time, as have structures which fall in the interface between combinatorics and other parts of mathematics. In the past year, we have obtained results about various algebraic structures, about combinatorial designs, about random structures, and about spaces of smooth, piecewise polynomial functions.

In work involving algebraic structures, we should mention again the work on orthogonal representations described in Section 1.3 and the work on matrices without 1 0 1 on the diagonal described in Section 1.2. Another example of such work involves extremal matrix problems. Let $W$ be a linear subspace of symmetric $n \times n$ matrices whose rank is at most $t$ and $L$ be an affine space of $n \times n$ matrices having rank at least $k$. Paper [62] obtains bounds on $\dim W$ if the underlying field has more than $t$ elements and bounds on $\dim L$ if the underlying field is algebraically closed. The latter result is applied to a problem of Valiant concerning permanents and determinants.

Combinatorial designs, in particular various kinds of balanced incomplete block designs, have played a role in the work described in Section 1.2 on competition graphs of acyclic digraphs with restricted indegrees and outdegrees. They have also played a role in the work on $\lambda$-hyperfactorizations of $K_{2n}$ described in that section. Designs also play a role in our study of projective planes. Paper [9] shows that in the Galois plane $PG(2,p^s)$, $p$ prime, there is a blocking set of size $2p^s$, with no lines having more than $p+1$ points in common with it. In paper [12], it is shown that if $PG(d,q)$ denotes the d-dimensional finite projective plane of order $q$, then given $1 \leq r$

$\leq s < d$ and $\epsilon > 0$, if $X \subset PG(d,q)$ contains $(1+\epsilon)q^s$ points, then the number of $r$-flats spanned by $X$ is at least a positive fraction of the number of $r$-flats in $PG(s+1,q)$.

An increasingly important theme in discrete mathematical research is to investigate random discrete structures of various kinds. The reason for the emphasis on random structures is in part because of their connections to probabilistic algorithms and in part because of their relevance in formulating models for applied problems. Moreover, sometimes a probabilistic approach can lead to useful results about inherently non-probabilistic problems. A few of our results are of a probabilistic nature. For instance, paper [52] shows that the second eigenvalue of a random $d$-regular graph is of the order of $\sqrt{d}$. This result was proved by a totally new method, quite different from the classical methods of random graphs. When it was later proved by classical methods, the proof was much more complicated. Another result about randomness is concerned with the collective coin flipping problem. A set of $n$ players wish to agree on the value of a single bit (0 or 1), which they will all accept as random. Each player has a fair coin at his disposal, which he flips privately. An obvious procedure is to have one of the players flip his coin and announce the outcome. Assuming that the designated player abides by the rules of the game, the outcome is an unbiased bit. However, if the player designated to flip the coin is dishonest, he can announce whatever bit value he chooses. Roughly speaking, the problem of <u>collective coin flipping</u> is to design a procedure for $n$ players to agree on a bit value which is robust in the sense that even in the presence of dishonest players who conspire to bias the coin, the outcome is unbiased or nearly unbiased. In the paper [3], we survey results about this problem viewed as a perfect information game. Viewed in this way, the collective coin flipping problem has connections with game theory, error-correcting codes, and extremal set theory and is an example of a more general question: How well can imperfect random sources simulate perfect random sources? The paper [3] surveys results about this question for other models of imperfect random sources. The paper [68] describes the most robust scheme known for the collective coin flipping problem. To give still a different example of the role of randomness in our research, we have presented a new method to analyze and solve the maximum satisfiability problem using a method which involves randomizing the Boolean variables. This method, written up in paper [13], is discussed in more detail in Section 4.2.

One of the themes of modern discrete mathematics and discrete optimization research is the investigation of the interface between these areas and other areas of mathematics. We have been investigating some problems of algebra, topology, and numerical analysis using combinatorial methods. Namely, we have been concerned with the general problem of finding efficient ways to represent all piecewise polynomial functions which are smooth of order $r$ and are of degree at most $m$, over a $d$-dimensional triangulated region in $d$-space. Such functions, often called <u>splines</u> or <u>finite elements</u>,

have application in computer graphics and surface modeling as well as in numerical analysis. We have been concerned with the problem of determining the dimensions of such spaces as well as computationally useful bases. In attacking this problem, we have used methods from combinatorics and commutative algebra similar to those used to study the numbers of faces of convex polytopes. We have related the study of the ring of continuous piecewise polynomials over a d-dimensional triangulation to the study of the face ring of the underlying simplicial complex and showed how this leads directly to a specification of the dimensions in question. In the past year, we have prepared four papers on this subject, papers [4,5,6,7]. Among the more important results of this work is the possibility that Groebner basis methods of computational commutative algebra might lead to effective means to compute dimensions and bases of spline spaces in large practical examples.

## 4. Computational Complexity and Efficient Algorithms

One of the primary directions of research in discrete mathematics today involves the complexity of algorithms. Our work on complexity has emphasized the foundations of computational complexity, the development of heuristic algorithms for obtaining approximate solutions to problems, and the development of on-line algorithms for the solution to problems where decisions must be made one at a time as data becomes available, rather than waiting for the entire problem to be laid out. In addition, throughout our research, we have been searching for efficient algorithms for solving problems, and many of these are referred to elsewhere in this report.

### 4.1. Foundations of Computational Complexity

Our work on foundations of computational complexity has spanned a variety of topics in theoretical computer science. For instance, in paper [50], we have studied the conjecture that, stated imprecisely, says that if you want to compute even a very simple function with a certain kind of boolean circuit (a bounded depth boolean circuit with gates with output functions depending only on a restricted sum of inputs), the circuit must be very large. We have proven this conjecture for a special case.

To give a second example, in paper [57] we have taken the position that communication is concerned with the question of how much information two processors need to exchange to compute a specified function that depends on both of their inputs. We develop a general framework for the study of a broad class of communication problems which can be looked at this way, and analyze such problems using combinatorial lattice theory.

To give a third example, in paper [23] we establish tradeoffs between the cost of queries and updates in various data structures.

## 4.2. Heuristics

As more and more problems are shown to be difficult, for instance by proving them to be NP-complete, there is coming to be an increasing emphasis on heuristic solutions. Heuristic algorithms are especially important in practice where there are many problems involving hundreds, thousands, even tens of thousands of variables. In such a case, we would like to elaborate a heuristic algorithm capable of very rapidly finding approximate solutions to large problems.

We have been working on heuristic algorithms for a number of combinatorial problems. In the past year, we have emphasized the Maximum Satisfiability Problem (MSP). In this problem, we seek the largest possible set of logical clauses from a given set which may be simultaneously satisfied. This problem contains the satisfiability problem and is NP-complete even when all clauses contain at most two literals. In paper [30], we study both old and new algorithms for MSP and Maximum 2-Satisfiability. In particular, we analyze the algorithms of Johnson [1974], Lieberherr [1982], Lieberherr and Specker [1981], and Poljak and Turzik [1982]. We propose an exact algorithm which can be used for problems of moderate size, as well as the specialization of two recent local search algorithmic schemes, the Simulated Annealing method of Kirkpatrick, Gelatt, and Vecchi [1983] and the Steepest Ascent Mildest Descent method. The resulting algorithms, which avoid being blocked as soon as a local optimum has been found, are shown empirically to be more efficient than the heuristics previously proposed in the literature.

In related work, we present in paper [13] a probabilistic model for MSP and apply probabilistic bounds to develop Branch-and-Bound type algorithms for their solution. (It is interesting to note that the probabilistic existence proof results in a deterministic algorithm.) The probability bounds prove the existence of some solutions having reasonable quality.

## 4.3. On-Line Algorithms

Many practical problems call for efficient solution algorithms for combinatorial problems. Most of the theory of combinatorial algorithms deals with problems which are completely specified in advance. In practical problems, we often need solution algorithms which are <u>on-line</u> in the sense that one is forced to make choices at the time data becomes available. A typical example of such a practical problem is the frequency assignment problem which is discussed in Section 1.1. One has to find an assignment of a frequency to a new transmitter before one knows all of the transmitters to which assignments must be made.

A general approach to on-line problems is to think of them as

sequential decisionmaking problems. There are two points of view: (a) Formulate a probabilistic model of the future and minimize the expected cost of future decisions; (b) compare an on-line decision strategy to the optimal off-line algorithm, one that works with complete knowledge of the future. The first is the approach taken for instance in the theory of Markov decision models. We have emphasized the second approach.

In this approach, one calculates the performance ratio, the ratio between the worst case value computed by an on-line algorithm and the theoretically optimal value. For instance, suppose $A$ is an on-line algorithm for coloring graphs. What this means is that a graph is presented one vertex at a time (with edges to previously presented vertices defined at that time). Then algorithm $A$ is supposed to choose a color for that vertex, knowing what it has previously done but not what the rest of the graph looks like. Let $O(G)$ be an order of presentation of $G$. Let $\chi_A(O(G))$ be the number of colors needed to color $G$ using algorithm $A$ if $O(G)$ is the order of presentation, and let $\chi_A(G)$ be the maximum $\chi_A(O(G))$ among all possible orders $O(G)$ in which $G$ could be presented. We are interested in the <u>performance ratio</u> $\chi_A(G)/\chi(G)$. Alternatively, we are sometimes interested in finding constants $\omega$ so that $\chi_A(G) - \omega\chi(G)$ is bounded for all $G$. Such a constant $\omega$ is called <u>a waste factor</u> for $A$. Then we seek $\omega(A)$, the infimum of all waste factors $\omega$ for $A$. This is called <u>the waste factor</u> for $A$.

We have shown in paper [59] that for every on-line algorithm $A$ and every graph $G$, $\chi_A(G)/\chi(G) \leq n/\log^* n$, where $n$ is the number of vertices of $G$ and $\log^* n$ is the number of times we must take the log of the log ... of $n$ before reaching a number less than or equal to 1. We have also shown that for every on-line algorithm, there is a graph $G$ so that $\chi_A(G)/\chi(G) \geq n/\log\log n$. The question remained whether or not the actual waste factor was sublinear, i.e., was it $o(n)$? We have now succeeded in showing this as well.

## 5. Applications of Discrete Mathematics in Decisionmaking

Problems involving complex choices are often most naturally formulated using discrete mathematics. The tools of discrete mathematics are widely used in the literature of individual or group decisionmaking, measurement and utility theory, and so on. (See references in our original proposal.) We have made progress using discrete techniques on several problems involving decisionmaking and have also found concepts from the theory of decisionmaking to be useful in attacking other problems of discrete mathematics.

### 5.1. Group Decisionmaking

Our major effort in group decisionmaking has involved various

results of a game-theoretic nature, in particular about multi-person games of the sort which arise in negotiation situations, in disarmament, in economic decisionmaking, and the like.

In game theory, there are several measures of power of a player in a group. We have studied two of these, the Shapley and Banzhaf measures. One important question in game theory is the question of how small the maximum (normalized) Banzhaf measure can be in a game with half of all coalitions winning. In work begun last year (paper [47]) we reduce this problem to one of estimating the maximum over $F \subseteq \{0,1\}^n$ of given size of the number of pairs $x,y$ in $F$ with $d(x,y) = d$, where $d(x,y)$ is the ordinary Hamming distance and $d$ is fixed in advance. The reduction depends on taking the Fourier transform of the characteristic vector of the set $X \subseteq \{0,1\}^n$ and then (reversing a standard technique) attempting to analyze this transform by studying an associated random walk. The advantage of the new formulation is that it is closer to standard extremal problems in combinatorial set theory, and hence might be more tractable.

The paper [48] is concerned with an idealized two-person game in which each player knows the other player's "hand." The results give an indication of how the "relative strength" of a hand determines the best strategy.

Other work on group decisionmaking which we have already discussed in Section 3.4 is our work on the collective coin flipping problem. This involves the design of a procedure whereby a group of $n$ decisionmakers can agree on the value of a single bit which they will all accept as random even in the presence of dishonest players.

## 5.2. Measurement and Decisionmaking

In our work on measurement and decisionmaking, we have worked on the meaningfulness of statements arising in combinatorial optimization and on a variety of clustering problems.

An area of considerable current interest in the theory of measurement is the theory of meaningful statements. Put briefly, a statement is called <u>meaningful</u> if its truth or falsity is independent of the particular versions of scales of measurement used. That is, its truth or falsity is not an artifact of the particular scales of measurement used. (See Roberts [1979a] for detailed definitions of concepts from the theory of measurement.) The theory of meaningfulness has had a wide variety of applications, including applications involving average performance measures for new technologies, importance ratings, indices of consumer confidence, psychophysics, social networks, and structural modeling for complex decisionmaking problems. See Roberts [1985] for a survey. Continuing work begun last year, we have looked at the limitations which the requirements of meaningfulness place on conclusions from combinatorial

optimization problems. The results show for instance that the statement "z is an optimal solution" can be meaningless for quadratic optimization even if the variables are measured on the very strong scale called a ratio scale. However, this statement is meaningful for linearly homogeneous objective functions under some reasonable assumptions. See paper [65].

Also falling into the general area of measurement and decisionmaking is the paper [66] which we have prepared which surveys the applications of graph theory and combinatorics in the biological and social sciences. In this paper, we discuss applications of the fundamental notion of semiorder which is critical in measurement theory and of the fundamental notion of social welfare function which is crucial in group decisionmaking. Also with decisionmaking applications in mind, we discuss problems of qualitative stability and applications of signed digraphs.

Much of measurement and decisionmaking begins with clustering or partitioning of alternatives into groups. Our work on measurement so far has concentrated to a large extent on clustering or partitioning problems. Clustering methods aim at finding within a given set of entities, subsets called clusters which are both homogeneous and well-separated. These concepts can be made precise in terms of dissimilarities between entities. The <u>split</u> of a cluster is the smallest dissimilarity between an entity in that cluster and one outside it. The paper [31] gives an $O(n^3)$ algorithm to determine the maximum sum of splits partitions into $M$ clusters for all $M$ between $N-1$ and 2. The paper [22] is a revised and significantly improved version of paper [31]. It introduces the concept of a dual graph of a dendrogram and solves the problem as a constrained longest path on this dual graph. The result is a $\theta(n^2)$ algorithm. The paper [10] also gives results on this problem. It is shown that there exists an optimal partition such that the intersection of the $j^{th}$ cluster with the convex hull of the first $j-1$ clusters is always empty.

## 5.3. Multiple Conclusion Logic

When a given signal can be interpreted as being the result of a variety of causes and a small number of tests have to be created to identify the exact cause of the signal, we have a typical instance of a multiple conclusion logic situation. Examples of such situations occur in medical decisionmaking, in searching and seeking in hazardous or nuclear or chemically toxic environments, in detecting enemy positions, in remote operations in space or underseas, and so on. This situation is especially important in the design of expert systems. A natural approach to such problems is based on Boolean methodology. The paper [18] uses partially defined Boolean functions to analyze multiple conclusion logic situations, in particular, cause-effect relationships. Procedures are provided to extrapolate from limited observations, concise and meaningful theories to explain

the effect under study and to prevent (or provoke) its occurrence. A follow-up to paper [18] is the paper [73], which looks at the same problem using methods in the spirit of regression analysis, implemented by ordinary linear programming, as opposed to the integer programming methods used in [18].

As we have noted, the work on multiple conclusion logic is relevant to the design of expert systems. For instance, Boolean methodology is useful in the design of "inference machines" for such systems, as was illustrated in work completed in an earlier year of this project - see Crama, et al. [1986]. Also, as the size of databases and knowledge bases in expert systems grows, the occurrence of inconsistencies becomes more and more likely. It is then desirable to restore consistency by relaxing as few logical conditions as possible. In the domain of propositional calculus, this problem corresponds to the maximum satisfiability problem for Horn formulas, a problem which has played a central role in the theory of Boolean functions. Our work on this problem has been described in Section 4.2.

## 6. New Operations Research Techniques for Large Scale Scheduling Problems

Since September 30, 1988, a group from RUTCOR has been working on developing new operations research techniques for dealing with large scale scheduling problems under a $100,000 supplement to RUTCOR's grant from AFOSR. The funds for this supplement were provided by the Military Airlift Command (MAC).

The central focus of this group's effort has been to determine models which as realistically as possible correspond to very large scale scheduling problems of interest to MAC, and to derive techniques to provide optimum or near optimum solutions for a variety of realistic objective functions.

RUTCOR's work on problems of MAC has involved basic research to formulate models for approaching various MAC questions, to develop techniques for handling MAC problems, and to develop and experiment with computational methods designed for implementation of some of the results. In addition, we have consulted with MAC on practical implementation problems.

The work for MAC has emphasized three problems which were recommended by MAC: The STORM I and STORM II models for routing aircraft on regularly scheduled MAC runs, the aircrew scheduling problem, and the single base aircrews staging problem. The work on these problems is described in the next three sections.

### 6.1. The STORM I and STORM II Models

The STORM I and STORM II linear programming models were developed by the CINCMAC Analysis Group for scheduling channel missions more efficiently. (STORM stands for Strategic Transport Optimal Routing Model.) These models were developed to save money by using fewer aircraft flying hours to airlift the same amount of cargo. The STORM I model has 3506 constraints and 4058 variables, while the STORM II model, which allows for transshipment, has 4204 constraints and 22,573 variables. Both models have proved extremely time consuming to solve with commercial simplex codes. David Shanno of RUTCOR has been working at the frontiers of research in this area by developing interior point methods for solving these models, with considerable practical success.

Numerical work over the past several years has provided substantial evidence that interior point algorithms for solving LP's outperform the classical simplex algorithm by a factor that grows rapidly with problem size. These interior point algorithms originated in the projective transformation methods for linear programming of Karmarkar in 1984.

Much of Shanno's research under his own AFOSR project during the past year was spent on separate implementations of two algorithms out of the group of logarithmic barrier algorithms, the dual affine algorithm and the primal-dual algorithm. His work has shown that all interior point methods fall into the general class of logarithmic barrier methods, of which the two implemented are the two of principal interest. This work has been documented in the report [70].

Past improvements in the primal-dual algorithm required considerable theoretical effort but still led to extraordinarily slow practical algorithms. Shanno has utilized the special feature of the primal-dual algorithm that an estimate of the decrease in the duality gap dependent upon the value of $u$ is available to devise an algorithm that has performed exceptionally well in practice. The preliminary study of this algorithm was partially supported under Shanno's AFOSR project and was written up in a report by McShane, et al. [1988]. Under MAC support, extensive additional testing was undertaken during the summer of 1988 and has been written up in report [14].

Implementation of the dual affine algorithm variant is described in report [60], which was also partly supported under the MAC funds. The performance of this algorithm has been so exceptionally good that recent work has concentrated on supercomputer implementations. Extensive research is currently under way on the CRAY XMP, YMP, and CRAY II. Results here are startling. For example, we can currently solve the STORM II model of MAC in 86 seconds on the CRAY II. This is much faster than the best simplex codes tailored to the CRAY. Indeed, Cray Research is so enthusiastic about the performance of this code that they have made available extensive free testing time on their computers in Minnesota and MAC has directly benefitted from extensive

free supercomputer testing of their models. A report documenting the implemented algorithm and the computational results is in preparation (report [61]).

To give more details about the performance of our codes on specific MAC models, we note that we have extensively tested both the STORM I and STORM II models, principally on the VAX8600, where we compared the performance of the interior point code to XMP, a standard linear programming code, and on the CRAY II, where we compared against LAMPS and LOPT, the two best simplex codes on the CRAY machines, both of which have been specifically tailored to the CRAYs. In all cases, the dual affine code was faster. On the VAX, we were approximately 10 times faster than XMP on STORM I. The advantage on STORM II was greater but not determined exactly, as XMP was terminated before optimality due to excessive time usage. On the CRAYs, the dual affine was twice as fast as LAMPS for STORM I, about 5 times faster than LAMPS for STORM II. LAMPS was faster than LOPT for these problems. Estimates of CRAY scientists are that the advantage of the dual affine can be enhanced by a factor of three by tailoring the software to CRAY architecture. Similar performance gains should be achievable for other vector machines. We have been working closely with MAC to illustrate the improved performance of our algorithms.

Several avenues of current research have been suggested by our work with the MAC models. First, we are examining the structure of the MAC constraint matrices to determine how best to implement linear algebra routines for interior point methods with these structures. More importantly, current research is under way to determine good heuristics to recover a near optimal integer feasible solution from the continuous optimal solution, which we think would be of major importance to MAC. This is an extremely difficult problem, but the structure of interior point algorithms suggests some promising heuristics which would have been difficult, if not impossible, to implement with a simplex code. This work is of such strong general interest that it will be the major reseach emphasis for further development of the dual affine codes for the next year.

### 6.2. The Aircrew Scheduling Problem

The Aircrew Scheduling Problem (ASP) is concerned with enhancing pilot satisfaction from assignments to particular flights while satisfying various feasibility constraints. This problem was the subject of a preliminary study by Albert Williams and Pierre Hansen of RUTCOR, who collaborated with Major William Carolan of the CINCMAC Analysis Group of MAC and with schedulers from various bases of the Air Force. The problem of keeping pilots happy is of vital importance to MAC, which estimates that the loss of each pilot represents an investment loss of more than a million dollars.

ASP is a complex and large-scale problem. We made a preliminary analysis of the problem, through meetings with Major Carolan at RUTCOR and at MAC, through the study of documents provided by MAC, and

through numerous interviews with pilots and schedulers. This analysis involved considerable effort at modelling of the problem. We determined that the following characteristics of ASP appear to be basic. First, it is complex, involving many actors, several possible objective functions, and many constraints. Second, ASP is partly defined in a fuzzy way, as pilot satisfaction, a psychological phenomenon which cannot be evaluated directly. Third, ASP is partly stochastic, as there are unscheduled flights and uncertain round-trip times on scheduled flights. Fourth, ASP involves go-nogo decisions, such as assigning a crew to a particular mission. (This means using 0-1 variables in the modelling rather than continuous variables.) Fifth, ASP is large-scale, larger than the scheduling problems solved to optimality in the current OR state of the art. Finally, ASP is a recurrent problem, solved manually each month. Any new system should improve over the actual ones, in terms of performance, flexibility, and user-friendliness.

Our preliminary analysis of ASP from a modelling point of view was followed by considerable effort at outlining the desirable characteristics of a computer-assisted scheduling system (CASS). We identified the following desirable characteristics of CASS. First, CASS should be a computer-assisted system, not an entirely automatic one. Second, it should prepare monthly schedules as well as modified schedules (or rolling schedules for the next 30 days). Third, CASS should use a flexible way to express preferences of the crew members. Fourth, CASS should use a heuristic algorithm to determine best schedules. Fifth, it should be usable as a simulation tool to explore policies. Sixth, it should alleviate as much as possible the administrative burden of schedulers and crew members. Finally, CASS should be flexible enough to adapt to different wings and to changing policies and conditions.

Having identified the desirable characteristics of CASS, we devoted considerable effort to outlining some basic principles upon which such a system might be built. We believe that the development of CASS should comprise first a prototype system which would be tested at one or two wings. If such a prototype system turned out to be promising, the task of completely realizing it as an industrial system would follow. This task would be much larger, particularly due to the large data bases involved, which would be accessed repeatedly and interactively, and which would need to be updated both temporarily and permanently.

The development of a prototype system would involve a large amount of basic research. We have outlined some of the steps needed to develop such a system. First, one would need to understand better the dimensions of pilot satisfaction (which may vary a lot from pilot to pilot according to familial and other personal preference characteristics). Second, one would need to derive a flexible system for obtaining and expressing pilot preferences. Third, one would need to derive a heuristic algorithm which would insure a high average level of pilot satisfaction while respecting all constraints on crew composition, availability, duties, etc. and keeping also a fairness

objective in view.

We have tentatively designed a bidding procedure which could play a key role in allowing flexible expression of pilot preference and crew scheduling, with both high average satisfaction and fairness.

We have proposed to MAC that a RUTCOR group, working in close collaboration with schedulers and operations researchers in the Air Force, be involved in the development of a prototype CASS. The RUTCOR group would focus on the fundamental modelling and mathematical issues involved, with the goal of producing a usable pilot software product. However, it would not aim to produce a complete realization of a commercial quality computer system. Such a system would have to be left to commercial software producers.

The results of our feasibility study on the Aircrew Scheduling Problem and a detailed proposal on the development of a pilot system were prepared as report [43]. A second report on this, [44], is being prepared.

### 6.3. The Single Base Aircrews Staging Problem

The Aircrews Staging Problem is concerned with the question of how many crews should be stationed in one given staging area (base) during a planned airlift operation. In general, the airlift operation evolves during a predetermined period of time (say, several weeks) and involves a given schedule of flights between several bases (origins and destinations) which serve to move cargo and personnel as required by the exercise. Notwithstanding the planning, the flows in this network are stochastic as a result of schedule changes during the operation which are due to factors such as changes in the priorities of missions, equipment failure, weather conditions, and so on. We have studied this problem for a single staging area, with the objective being to develop methods and algorithms for achieving two goals. The primary goal is to minimize the number of crews stationed at the staging area so as to achieve a predetermined very small probability of aircraft delay. The secondary goal is to minimize crew discontent due to unnecessary waiting (legal periods).

Preliminary analysis of the Single Base Aircrews Staging Problem (SBASP) was carried out by Benjamin Avi-Itzhak of RUTCOR, in collaboration with Major Carolan of MAC. This analysis at first involved various attempts to formulate mathematical models. We came to the conclusion that SBASP could be formulated as an inventory planning and control process and that it should be handled through the development of two systems: A staging planning system and a staged crews management system.

We did some research work on the planning problem and found out that early planning (1-3 months before the exercise) can be aided by a model based on aircraft arrivals behaving as a renewal process. At this phase of the planning there is only very rough information

available and the renewal process assumption requires very little information. However, at the final planning phase, when flight schedules are available, there is a need for an algorithm of a very general nature that will track the schedule and calculate the number of needed crews at each point in time during the whole exercise. We did some research work developing the analysis and mathematical results for the renewal process approach, including working out some numerical examples. This work is documented in the report [2]. One interesting result here was that the criterion of average aircraft delay is not very meaningful and it may be more desirable to minimize the number of staged crews subject to a given predetermined probability of delay.

Professor Avi-Itzhak sent a preliminary copy of report [2] to MAC and then flew to St. Louis to discuss the results with Major Carolan and others involved in the SBASP. They had adopted his suggested approach but were trying to solve the final planning problem by the use of a brute force simulation. He provided Major Carolan with an outline of a numerical algorithm which is orders of magnitude superior to brute force simulation and much more reliable.

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Note: References to papers with numbers, e.g., [3], refer to papers supported by the grant and prepared during the period September 30, 1987 to September 30, 1988. These papers are listed in the next section.

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A RUTCOR Project on Discrete Applied Mathematics

Grant Number AFOSR 85-0271

List of Publications

September 30, 1987-September 30, 1988

1. N. Alon, J. Kahn, and P. Seymour, "Large Induced Degenerate Subgraphs," *Graphs and Combinatorics*, 3 (1987), 203-211.

2. Avi-Itzhak, B., "On the Aircraft Crews Staging Problem: A Preliminary Study of the Single Staging Area Case," preliminary draft, July 6, 1988. Revised draft, September 29, 1988.

3. Ben Or, M., Linial, N., and Saks, M., "Collective Coin Flipping and other Models of Imperfect Randomness," RUTCOR Research Report RRR-44-87, December 1987. In *Colloq. Math. Soc. Janos Bolyai (Combinatorics), Eg. Hung.*, 1987, 75-112.

4. Billera, L.J., "A Dimension Series for Multivariate Splines," *Discrete and Computational Geometry*, to appear.

5. Billera, L.J., "Combinatorial Decomposition and Normal Form of the Harmonic Oscillator," *Proc. of the Koninklijke Nerlandse Akademie von Wetenschappen*, Series A, 91 (1988).

6. Billera, L.J., "Groebner Basis Methods for Multivariate Splines," prepared for the *Proceedings of the Oslo Conference on Computer-aided Geometric Design*, June 1988.

7. Billera, L.J., "Homology of Smooth Splines: Generic Triangulations and a Conjecture of Strang," *Trans. Amer. Math. Soc.*, 310, to appear.

8. Billera, L.J., and Haas, R., "The Dimension and Bases of Divergence-Free Splines," submitted for publication.

9. Boros, E., "PG(2,$p^s$) > 2 has Property B(P+2)," RUTCOR Research Report RRR-33-87, October 1987. In *Ars Combinatoria*, 25 (1988), 111-113.

10. Boros, E., and Hammer, P.L., "On Clustering Problems with Connected Optima in Euclidean Spaces," RUTCOR Research Report RRR-33-88, March 1988. To appear in B. Bollobas (ed.), *Proc. Conf. Comb.: Erdos 75*, Cambridge, 1988.

11. Boros, E., Jungnickel, D., and Vanstone, S.A., "The Existence of Non-trivial Hyperfactorizations of $K_{2n}$," RUTCOR Research Report RRR-13-88, March 1988. To appear in *Combinatorica*.

12. Boros, E., and Meshulam, R., "On the Number of Flats Spanned by a Set of Points in PG(d,q)," RUTCOR Research Report RRR-19-88, March 1988. To appear in *Annals of Discr. Math*.

13. Boros, E., and Prekopa, A., "Probabilistic Bounds and Algorithms for the Maximum Satisfiability Problem," RUTCOR Research Report RRR-17-88, March 1988. To appear in *Math. of Operations Research*.

14. Choi, I.C., Monma, C.O., and Shanno, D., "Further Algorithmic Development of a Primal-Dual Interior Point Method," RUTCOR Research Report RRR-60-88, in press. Submitted to *ORSA J. on Computing*.

15. Crama, Y., and Hammer, P.L., "Bimatroidal Independence Systems," RUTCOR Research Report RRR-25-88, June 1988.

16. Crama, Y., and Hammer, P.L., "Recognition of Quadratic Graphs and Adjoints of Bidirected Graphs," RUTCOR Research Report RRR-8-88, March 1988.

17. Crama, Y., and Hammer, P.L., "A Characterization of a Cone of Pseudo-Boolean Functions via Supermodularity-type Inequalities," RUTCOR Research Report RRR-41-1988, August 1988.

18. Crama, Y., Hammer, P.L., and Ibaraki, T., "Cause-Effect Relationships and Partially Defined Boolean Functions," RUTCOR Research Report RRR-39-88, August 1988.

19. Crama, Y., Hansen, P., and Jaumard, B., "The Basic Algorithm of Pseudo-Boolean Programming Revisited," in preparation.

20. Dearing, P.M., Hammer, P.L., and Simeone, B., "Boolean and Graph Theoretic Formulations of the Simple Plant Location Problem," RUTCOR Research Report RRR-3-88, January 1988.

21. de Werra, D., and Roberts, F.S., "On the Use of Augmenting Chains in Chain Packings," RUTCOR Research Report RRR-46-88, September 1988. To appear in *Discrete Applied Math*.

22. Frank, O., Hansen, P., and Jaumard, B., "Maximum Sum of Splits Clustering," *J. of Classification*, to appear.

23. Fredman, M., and Saks, M., "The Cell-Probe Complexity of Dynamic Data Structures," in preparation.

24. Furedi, Z., and Kahn, J., "On Dimension vs. Size," *Order*, 5 (1988), 17-20.

25. Furedi, Z., Kahn, J., and Kleitman, D.J., "Sphere Coverings of the Hypercube with Incomparable Centers," *Discrete Math.*, to appear.

26. Hammer, P.L., and Kalantari, B., "A Bound on the Roof-duality Gap," RUTCOR Research Report RRR-46-87, December 1987.

27. Hammer, P.L., and Liu, Y., "0-1 Programming and Order Relations, Part I," *J. Math. Res. & Expos.*, 8 (1988), 315-325 (in Chinese).

28. Hammer, P.L., and Liu, Y., "0-1 Programming and Order Relations, Part II," *J. Math. Res. & Expos.*, 8 (1988), 481-490 (in Chinese).

29. Hanjoul, K., Hansen, P., Peeters, D., and Thisse, J., "Uncapacitated Plant Location under Alternative Special Price Policies," *Management Sci.*, to appear.

30. Hansen, P., and Jaumard, B., "Algorithms for the Maximum Satisfiability Problem," RUTCOR Research Report RRR-43-87, November 1987.

31. Hansen, P., and Jaumard, B., "Maximum Sum of Splits Clustering," RUTCOR Research Report RRR-11-88, March 1988. (Revised: See paper 21.)

32. Hansen, P., Jaumard, B., and Lu, S., "A Framework for Algorithms in Globally Optimal Design," *ASME Trans. on Mechanisms, Transmissions, and Automation in Design*, to appear.

33. Hansen, P., Jaumard, B., and Lu, S., "An Automated Procedure for Globally Optimal Design," *ASME Trans. on Mechanisms, Transmissions, and Automation in Design*, to appear.

34. Hansen, P., Jaumard, B., and Lu, S., "Global Minimization of Univariate Functions by Sequential Polynomial Approximation," *Int. J. of Computer Math.*, to appear.

35. Hansen, P., Jaumard, B., and Lu, S., "On the Number of Iterations of Piyavskii's Global Optimization Algorithm," RUTCOR Research Report RRR-29-88, June 1988.

36. Hansen, P., Jaumard, B., and Lu, S., "Some Further Results on Monotonicity in Globally Optimum Design," *ASME Trans. on Mechanisms, Transmissions, and Automation in Design*, to appear.

37. Hansen, P., and Kuplinsky, J., "The Smallest Hard-to-Color Graph," RUTCOR Research Report RRR-5-88, February 1988.

38. Hansen, P., and Labbe, M., "Algorithms for Voting and Competitive Location on a Network," RUTCOR Research Report RRR-14-86, August 1986. To appear in *Transportation Science*.

39. Hansen, P., and Labbe, M., "The Continuous p-Median of a Network," to appear in *Networks*.

40. Hansen, P., Labbe, M., and Wendell, R.E., "Sensitivity Analysis in Multiple Objective Linear Programming," to appear in *Eur. J. of Operational Res.*

41. Hansen, P., Lu, S., and Simeone, B., "On the Equivalence between Paved-Duality and Standard Linearization in Nonlinear 0-1 Optimization," to appear in *Discrete Applied Math.*

42. Hansen, P., and Moon, D., "Dispersing Facilities on a Network," RUTCOR Research Report RRR-52-88, October 1988.

43. Hansen, P., and Williams, A., "Computer-assisted Aircrew Scheduling System: A Feasibility Study," August 9, 1988.

44. Hansen, P., and Williams, A., "On the Aircrew Scheduling Problem (Preliminary Title)," in preparation.

45. Hansen, P., and Zheng, M., "Bounds on the Order of Graphs with Given Size and Stability Number," RUTCOR Research Report RRR-42-88, September 1988.

46. Hefner, K.A.S., Jones, K.F., Kim, S., Lundgren, J.R., and Roberts, F.S., "(i,j) Competition Graphs," RUTCOR Research Report RRR-14-88, March 1988.

47. Kahn, J., Kalai, G., and Linial, N., "The Influence of Variables on Boolean Functions," in preparation. (Extended Abstract: The Influence of Variables on Boolean Functions, appeared in $29^{th}$ *FOCS, IEEE*, 1988.)

48. Kahn, J., Lagarias, J., and Witsenhausen, H., "Single Suit Two Person Card Play II," *Order*, 5 (1988), 45-60.

49. Kahn, J., and Linial, N., "Balancing Extensions via Brunn-Minkowski," submitted.

50. Kahn, J., and Meshulam, R., "On mod p Transverals," in preparation.

51. Kahn, J., and Seymour, P., "The Fractional Erdos-Faber-Lovasz Conjecture," *Combinatorica*, to appear.

52. Kahn, J., and Szemeredi, E., "The Second Eigenvalue of a Random Regular Graph," in preparation.

53. Kim, S., "Competition Graphs and Scientific Laws for Food Webs and Other Systems," Ph.D. Thesis, Department of Mathematics, Rutgers University, New Brunswick, N.J., October 1988.

54. Kim, S., McKee, T., McMorris, F., and Roberts, F.S., "p-Competition Graphs," in preparation.

55. Kim, S., and Roberts, F.S., "On Opsut's Conjecture for the Competition Number," in preparation.

56. Kim, S., Roberts, F.S., and Seager, S., "On 1 0 1-clear (0,1) Matrices and the Double Competition Number of Bipartite Graphs," in preparation.

57. Lovasz, L., and Saks, M., "Lattices, Moebius Functions, and Communication Complexity," *Proc.* 29$^{th}$ *FOCS*, 1988, 81-90. Expanded version in preparation.

58. Lovasz, L., Saks, M., and Schrijver, A., "Orthogonal Representations and Connectivity of Graphs," RUTCOR Research Report RRR-45-87, December 1987. To appear in *J. Lin. Alg. Appl.*

59. Lovasz, L., Saks, M., and Trotter, W.T., "An Online Algorithm for Graph Coloring with Sublinear Waste," submitted.

60. Marsten, R.E., Saltzman, M.J., Shanno, D.F., Pierce, G.S., and Ballintijn, J.F., "Implementation of a Dual Affine Interior Point Algorithm for Linear Programming," RUTCOR Research Report RRR-44-88, August 1988. Submitted to *ORSA J. on Computing*.

61. Marsten, R.E., and Shanno, D.F., "The Dual Affine Algorithm on Supercomputers," in preparation.

62. Meshulam, R., "On Two Extremal Matrix Problems," RUTCOR Research Report RRR-2-88, January 1988. To appear in *Lin Alg. & Appl.*

63. Peled, U.N., and Srinivasan, M.K., "The Polytope of Degree Sequences," RUTCOR Research Report RRR-47-87, December 1987.

64. Roberts, F.S., "From Garbage to Rainbows: Generalizations of Graph Coloring and their Applications," RUTCOR Research Report RRR-36-88, July 1988. To appear in Y. Alavi, G. Chartrand, O.R. Oellermann, and A.J. Schwenk (eds.), *Proceedings of the Sixth International Conference on the Theory and Applications of Graphs*, Wiley, New York.

65. Roberts, F.S., "Meaningfulness of Conclusions from Combinatorial Optimization," RUTCOR Research Report RRR-45-88, September 1988. Submitted for publication.

66. Roberts, F.S., "Seven Fundamental Ideas in the Application of Combinatorics and Graph Theory in the Biological and Social Sciences," in preparation.

67. Roberts, F.S., "Final Report to MAC: New Operations Research Techniques for Large Scale Scheduling Problems," preliminary draft, October 1988.

68. Saks, M., "A Robust Non-cryptographic Coin Flipping Scheme," to appear in *SIAM J. on Discr. Math*.

69. Saks, M., and Werman, M., "Computing Majority by Comparisons," submitted.

70. Shanno, D.F., and Bagchi, A., "A Unified View of Interior Point Methods for Linear Programming," RUTCOR Research Report RRR-35-88, July 1988. Submitted to *Annals of Oper. Res*.

71. Tesman, B., "Vertex List T-Colorings of Graphs," mimeographed, 1989.

72. Tesman, B., Ph.D. Thesis, Department of Mathematics, Rutgers University, New Brunswick, N.J., in preparation.

73. Williams, A.C., "Extrapolation of Partially Defined Boolean Functions by Linear Programming," RUTCOR Research Report RRR-40-88, August 1988.

74. Williams, A.C., "Marginal Analysis for Linear Programming Models," RUTCOR Research Report RRR-42-87, November 1987.

75. Williams, A.C., "The Separation Problem: Linearization and the Equivalence of all Degree 2 Binary Optimization Problems," RUTCOR Research Report RRR-21-88, April 1988.

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Lectures Delivered
September 30, 1987-September 30, 1988

Peter L. Hammer

"Order Relations of Variables in 0-1 Programming," Mini-conference on Discrete Mathematics, Clemson University, Clemson, South Carolina, November 1987.

"Product Form Parametric Representation of the Solutions of a Quadratic Boolean Equation," Fifth Conference on Mathematical Programming, Oberwolfach, January 1988.

"From Linear Separability to Unimodality," Cambridge Combinatorial Conference, England, March 1988.

"0-1 Programming and Order Relations," Partially Ordered Sets Meeting, Oberwolfach, April 1988.

"Stability in Circular-arc Graphs," University of Delaware, April 1988.

"More Characterizations of Triangulated and Cotriangulated Graphs," EURO IX/TIMS XXVIII, Paris, July 1988.

"A Bound on the Roof Duality Gap," Workshop on Mathematical Programming, Rio de Janeiro, July 1988.

"Boolean and Graph-theoretic formulations of the Simple Plant Location Problem," Simon Fraser University, Vancouver, British Columbia, August 1988.

"Cause-Effect Relationships and Partially Defined Boolean Functions," 13th International Symposium on Mathematical Programming, Tokyo, August 1988.

"A Characterization of a Cone of Pseudo-Boolean Functions via Supermodularity-type Inequalities," Mathematical Programming Days, Kyoto, August 1988.

"Order Relations of Variables in 0-1 Programming," M.I.T., Cambridge, MA, September 1988.

"0-1 Programming and Order Relations," Carnegie Mellon University, Pittsburgh, PA, September 1988.

Fred S. Roberts

"Applications of Intersection Graphs," invited minisymposium talk, SIAM National Meeting, Denver, Colorado, October 1987.

"Meaningless Statements, Matching Experiments, Generalized Fibonacci Sequences, and Colored Digraphs," colloquium talk, University of Colorado, Denver, October 1987.

"Meaningless Statements, Matching Experiments, Generalized Fibonacci Sequences, and Colored Digraphs," seminar talk, Northeastern University, Boston, MA, December 1987.

"The One-Way Street Problem," colloquium talk, Worcester Polytechnic Institute, Worcester, MA, December 1987.

"Seven Fundamental Ideas in the Applications of Combinatorics and Graph Theory to the Biological and Social Sciences," invited two-hour lecture at Workshop on Applications of Combinatorics and Graph Theory to the Biological and Social Sciences, Institute for Mathematics and its Applications, University of Minnesota, Minneapolis, January 1988.

"Meaningless Statements, Matching Experiments, and Colored Digraphs," invited lecture at Workshop on Applications of Combinatorics and Graph Theory to the Biological and Social Sciences, Institute for Mathematics and its Applications, University of Minnesota, Minneapolis, January 1988.

"Measurement, Utility, and Decisionmaking." A series of 14 lectures presented to Le Troisieme Cycle Romande in Operations Research, Grimentz, Switzerland, March 1988 (for operations research faculty and graduate students in French-speaking Switzerland).

Titles of Individual Lectures:

"Seven Fundamental Ideas in the Mathematics of the Social and Biological Sciences, Part I"
"Seven Fundamental Ideas in the Mathematics of the Social and Biological Sciences, Part II"
"The One-Way Street Problem"
"Introduction to the Theory of Measurement, Part I"
"Introduction to the Theory of Measurement, Part II"
"Structural Modeling"
"Introduction to the Theory of Measurement, Part III"
"Introduction to the Theory of Measurement, Part IV"
"From Rainbows to Ham and Cheese Sandwiches: T-Colorings of Graphs and the Channel Assignment Problem"
"Meaningless Statements, Part I"
"Meaningless Statements, Part II"
"Generalized Competition Graphs and their Applications"
"On the Possible Scientific Laws (On the Possible Merging Functions"

"Uniqueness in Finite Measurement Structures (Some New Ideas in Combinatorics and Number Theory)"

"Generalized Competition Graphs and their Applications," seminar at Miami University, Oxford, Ohio, April 1988.

"The One-Way Street Problem," colloquium talk at Miami University, Oxford, Ohio, April 1988.

Banquet Address, Department of Mathematics and Statistics, Miami University, Oxford, Ohio, April 1988.

"Graphs, Garbage, and a Pollution Solution," Princeton High School, May 1988.

"From Garbage to Rainbows," invited address to Graph Theory Day Conference, New York Academy of Science, New York, May 1988.

"Applications of Discrete Mathematics," series of six lectures presented to Mathematics Association of Two Year Colleges of New Jersey, Princeton, May 1988.

Titles of Individual Talks:

"Applications of Graph Coloring"
"Applications of Eulerian Chains and Paths"
"Applications of Simple Counting Rules"
"The One-Way Street Problem"
"Balance in Small Groups"
"Pulse Processes and Complex Decisionmaking Problems"

"From Garbage to Rainbows," invited plenary address to International Conference on Graph Theory and its Applications," Kalamazoo, Michigan, May 1988.

"On the Possible Merging Functions," invited hour talk at Workshop on Measurement Theory, Center for Advanced Study in the Behavioral Sciences, Palo Alto, California, June 1988.

"Finite Uniqueness Problems," invited hour talk at Workshop on Measurement Theory, Center for Advanced Study in the Behavioral Sciences, Palo Alto, California, June 1988.

"Applications of Discrete Mathematics," series of four lectures presented to Workshop of Consortium for Mathematics and its Applications, Boston, MA, June 1988.

Titles of individual Talks:

"Graph Coloring"
"Eulerian Chains and Paths"
"Simple Counting Rules"
"One-Way Streets"

"From Garbage to Rainbows," Douglass High School Institute for Science and Math, Douglass College, July 1988.

Eight Lecture Minicourse "Applications of Graphs and Relations," Allegheny Mountain Section, Mathematical Association of America, Allegheny College, Meadville, PA, July 1988.

Titles of Individual Talks:

"Applications of Graph Coloring"
"T-Colorings of Graphs"
"Applications of Eulerian Chains and Paths"
"Competition Graphs and their Applications"
"Relational Systems and the Theory of Measurement"
"Meaningless Statements"
"Representation and Uniqueness Theorems"
"The Possible Merging Functions"

"Applications of Discrete Mathematics," series of six lectures presented to Mathematics Association of Two Year Colleges of New Jersey, Princeton, September 1988.

Titles of Individual Talks:

"Intersection Graphs and their Applications"
"T-Colorings of Graphs"
"Competition Graphs and their Applications"
"Applications of Generating Functions"
"Group Decisionmaking: Arrow's Impossibility Theorem"
"Group Decisionmaking: Means and Medians"

Louis Billera

"Algebraic Methods for Multivariate Splines," Royal Institute of Technology, December 1987.

"Algebraic Methods for Multivariate Splines," Hebrew University of Jerusalem, January 1988.

"Algebraic Methods for Multivariate Splines," Institute for Mathematics and its Applications, University of Minnesota, Minneapolis, March 1988.

"Algebraic Methods for Multivariate Splines," SUNY at Binghamton, April 1988.

"Algebraic Methods for Multivariate Splines," Courant Institute, NYU, April 1988.

"Algebraic Methods for Multivariate Splines," University of Oslo, June 1988.

"Algebraic Methods for Multivariate Splines," Univesity of Augsburg, June 1988.

"Algebraic Methods for Multivariate Splines," IBM Yorktown Heights, July 1988.

Endre Boros

"On the Number of Flats Spanned by a Set of Points in PG(d,q)," Combinatorics '88, Ravello, Italy, May 1988.

"On a Lemma of Segre," University of Delaware, April 1988.

Pierre Hansen

"Clustering Algorithms," Institute for Mathematics and its Applications, University of Minnesota, Minneapolis, January 1988.

"An Analytical Approach to Global Optimization," University of Delaware, April 1988.

"Dispersing Facilities on a Network," ORSA/TIMS national meeting, Washington, D.C., April 1988.

"Algorithms for Feasible Designs," ORSA/TIMS national meeting, Washington, D.C., April 1988.

"Maximum Sum of Splits Clustering," ORSA/TIMS national meeting, Washington, D.C., April 1988.

"A Hard to Color Graph," Advanced Research Institute in Discrete Applied Mathematics, Rutgers, May 1988.

"An Analytical Approach to Global Optimization," a 90-minute research review, EURO IX, Paris, July 1988.

"Two Clustering Problems," EURO IX, Paris, July 1988.

"Some Exact Clustering Algorithms," Institute of Applied Mathematics, Beijing, China, August 1988.

"Recent Applications of Operations Research," Institute of System Science, Beijing, China, August 1988.

"Applied Graph Theory," 30 lectures at Academia Sinica, Beijing, China, August 1988.

"An Analytical Approach to Global Optimization," Mathematical Programming Society, Tokyo, August 1988.

"Probabilistic Logic and Pseudo-Boolean Programming," Mathematical Programming Society, Tokyo, August 1988.

Jeffry Kahn

"Fractions of Matchings and Fractional Matchings," M.I.T., Cambridge, MA, November 1987.

"On Fractional Matchings," IBM Research, Almaden, January 1988.

"Rebalancing Poset Extensions," Institute for Mathematics and its Applications, University of Minnesota, Minneapolis, March 1988.

"Fourier Analysis of a Problem on Finite Sets," Meeting on Posets, Oberwolfach, Germany, April 1988.

Roy Meshulam

"A Problem on Permanents and Determinants," M.I.T., February 1988.

"Linear Spaces of Matrices," Arizona State University, March 1988.

Michael Saks

"A Search Problem Related to Branch and Bound Procedures," Simon Fraser University, March 1988.

"A Search Problem Related to Branch and Bound Procedures," University of Arizona, April 1988.

"Local Management of a Global Resource in a Communication Network," University of California, San Diego, April 1988.

"Lattices, Moebius Functions, and Communication Complexity," University of Toronto, August 1988.

David Shanno

"An Implementation of a Primal-Dual Interior Point Method for Linear Programming," Conference on Optimization at Mathematical Research Institute, Oberwolfach, January 1988.

"An Implementation of a Primal-Dual Interior Point Method for Linear Programming," ORSA/TIMS national meeting, Washington, D.C., April 1988.

"A Unified View of Interior Point Methods for Linear Programming," Workshop on Optimization on Supercomputers, University of Minnesota, May 1988.

"An Implementation of a Primal-Dual Interior Point Method for Linear Programming," Mathematical Programming Society Symposium, Tokyo, August 1988.

Participants in "A RUTCOR Project on Discrete Applied Mathematics"

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